DISSIPATIVE SPECTRAL TRANSFERS OF TURBULENT PLASMA PULSATIONS

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We discuss the excitation of low-frequency ($\omega^s \ll \nu_e m_e/m_i$) acoustic vibrations by a beam of transverse (rf) waves. It is found that under certain conditions the dispersion (and not simply the excitation increment) of the low-frequency acoustic vibrations is uniquely connected with the rf wave energy density.

1. It is well known that plasma turbulence spectra are determined by the intensity of the transformation along the spectrum of the energy of the turbulent pulsations excited as a result of plasma instability. If the turbulence is quasistationary and the characteristic lifetime of the stationary turbulence is much greater than the pairwise collision frequencies, then the interaction of the turbulent pulsations may depend very significantly on the pairwise collisions of the particles. A calculation was made in [1] of the effectiveness of such interactions under conditions when the difference of the frequencies of the two interacting pulsations is much greater than $\nu_e m_e/m_i$ (m_e and m_i are respectively the electron and ion masses; the plasma is fully ionized; and ν_e is the electron collision frequency between the electrons and ions of the plasma). The more exact criteria necessary for applicability of the results of [1] have the form (violation of the first condition (1.1) does not lead to any significant change of the results of [1], leaving them valid in order of magnitude):

$$\left|\frac{\omega_1 - \omega_2}{k - v_{Te}}\right|^2 \gg \frac{m_e}{m_i}, \qquad |\omega_1 - \omega_2| \gg \nu_e \frac{m_e}{m_i} \tag{1.1}$$

The purpose of the present study is to analyze the nonlinear interactions and characteristic times for the spectral transfers under conditions when (1.1) is violated. It will be shown that under these conditions kinetic excitation of low-frequency vibrations by the high-frequency (as a result of decay type processes) is not possible, and hydrodynamic excitation occurs only under conditions of very narrow spectra of the high-frequency vibrations. However, in this frequency region a new specific form of instability, associated with the dissipative nature of the process, is possible.

In contrast with high-frequency wave transfers under the conditions (1.1), the spectral transfers examined below are dissipative, i.e., the real nonlinear corrections to the frequency may be significantly smaller than the imaginary corrections.

In the region in question the thermal motion of the particles affects only the spectra of the turbulent pulsation frequencies (but not their interaction); therefore it is possible to use the two-fluid hydrodynamic equations [2] to describe this interaction. It will be shown that the hydrodynamic equations make it possible to obtain the results of [1] under the conditions (1.1) if the friction force between the electrons and ions for the high-frequency vibrations is considered equal to ($-m_e n_e \nu_e U$) rather than ($-0.51 m_e n_e \nu_e U$). This is easy to understand if we consider that the coefficient 0.51 appears only under conditions of frequency collisions $\omega \ll \nu_e$, while under conditions of weak encounters direct calculation of the friction force yields ($-m_e n_e \nu_e U$). The system of equations written out below, taking this circumstance into account, can be considered semi-phenomenological. The justification for its use is confirmed by kinetic calculations [1].

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2. We write out this system

$$\frac{\partial n_e}{\partial t} + \operatorname{div}(n_e \mathbf{V}_e) = 0, \qquad \frac{\partial n_i}{\partial t} + \operatorname{div}(n_i \mathbf{V}_i) = 0 \tag{2.1}$$

$$m_e n_e \left[\frac{\partial}{\partial t} + \left(\mathbf{V}_e \frac{\partial}{\partial \mathbf{r}}\right)\right] V_{e,\alpha} = -\frac{\partial}{\partial x_\alpha} n_e T_e - \frac{\partial \pi_{e,\alpha\beta}}{\partial x_\beta} - e n_e \left(E_\alpha + \frac{1}{c}[\mathbf{V}_e \mathbf{H}]_\alpha\right) + R_\alpha \tag{2.2}$$

$$m_i n_i \left[\frac{\partial}{\partial t} + \left(\mathbf{V}_i \frac{\partial}{\partial \mathbf{r}}\right)\right] V_{i,\alpha} = -\frac{\partial}{\partial x_\alpha} n_i T_i - \frac{\partial \pi_{i,\alpha\beta}}{\partial x_\beta} + e n_i \left(E_\alpha + \frac{1}{c}[\mathbf{V}_i \mathbf{H}]_\alpha\right) - R_\alpha \tag{2.3}$$

$$\frac{3}{2} n_e \left[\frac{\partial}{\partial t} + \left(\mathbf{V}_e \frac{\partial}{\partial \mathbf{r}}\right)\right] T_e + n_e T_e \operatorname{div} \mathbf{V}_e = -\operatorname{div} \mathbf{q}_e - \pi_{e,\alpha\beta} \frac{\partial V_{e,\alpha}}{\partial x_\beta} + Q_e \tag{2.4}$$

$$\frac{3}{2} n_i \left[\frac{\partial}{\partial t} + \left(\mathbf{V}_i \frac{\partial}{\partial \mathbf{r}}\right)\right] T_i + n_i T_i \operatorname{div} \mathbf{V}_i = -\operatorname{div} \mathbf{q}_i - \pi_{i,\alpha\beta} \frac{\partial V_{i,\alpha}}{\partial x_\beta} + Q_i \tag{2.5}$$

$$\mathbf{R} = \mathbf{R}_U + \mathbf{R}_T, \qquad \mathbf{q}_e = \mathbf{q}_U{}^e + \mathbf{q}_T{}^e, \qquad \mathbf{U} = \mathbf{V}_e - \mathbf{V}_i$$

Here $\mathbf{R}_U$ is the friction force, $\mathbf{R}_T$ is the thermal force, m_e, m_i are the masses, n_e, n_i are the concentrations, and T_e, T_i are the temperatures of the electrons and ions, respectively:

$$\mathbf{R}_T = -0.71\, n_e \frac{\partial T_e}{\partial \mathbf{r}}, \qquad \mathbf{R}_U = -n_e m_e \nu_e \mathbf{U} \begin{cases} 0.51 & \text{for} \quad \omega \ll \nu_e \\ 1.0 & \text{for} \quad \omega \gg \nu_e \end{cases} \tag{2.6}$$

$$\mathbf{q}_U{}^e = 0.71\, n_e T_e \mathbf{U} \qquad \mathbf{q}_T{}^e = -3.16 \frac{n_e T_e}{m_e \nu_e} \frac{\partial T_e}{\partial \mathbf{r}}, \qquad \mathbf{q}_i = -3.9 \frac{n_i T_i}{m_i \nu_i} \frac{\partial T_i}{\partial \mathbf{r}}$$

$$Q_e = -(\mathbf{RU}) - Q_i, \qquad Q_i = 3 \frac{m_e}{m_i} n_e \nu_e (T_e - T_i)$$

$$\pi_{e,\alpha\beta} = -0.73 \frac{n_e T_e}{\nu_e} W_{\alpha\beta}^{(e)}, \qquad \pi_{i,\alpha\beta} = -0.96 \frac{n_i T_i}{\nu_i} W_{\alpha\beta}^{(i)}$$

$$W_{\alpha\beta} = \frac{\partial V_\alpha}{\partial x_\beta} + \frac{\partial V_\beta}{\partial x_\alpha} - \frac{2}{3} \delta_{\alpha\beta} \operatorname{div} \mathbf{V}$$

The quantities ν_e and ν_i are the characteristic collision frequencies of the electrons and ions with the other particles (electrons and ions) of the plasma, which depend in a known fashion on the temperature ($\sim T_\alpha^{-3/2}$) and density of the plasma.

We shall use this system of equations to study nonlinear interactions of high-frequency pulsations

$$\omega^+ \gtrsim \omega_{0e}, \quad \omega_{0e} = (4\pi n_e e^2 / m_e)^{1/2}$$

In the nonlinear polarizabilities of the plasma we shall take into account terms which include no higher than the second power of ω^+ in the denominator. We expand all quantities in powers of the electric field intensity up to and including terms of third order in E

$$V = \sum_{j=1} V^{(j)}, \qquad n = n_0 + \sum_{j=1} n^{(j)}, \qquad T = T_0 + \sum_{j=1} T^{(j)} \tag{2.7}$$

$$V^{(j)} \sim (E)^j, \qquad n^{(j)} \sim (E)^j, \qquad T^{(j)} \sim (E)^j$$

Here $n_0 = \langle n \rangle$, $T_0 = \langle T \rangle$ are the average values of the temperature and density in the turbulent plasma. We can use (2.7) if

$$(n^{(j)} / n_0), \ (T^{(j)} / T_0), \ \ldots \ll 1$$

We note immediately that the presence of high-frequency turbulent pulsations affects the average plasma particle distribution function and leads to the so-called collisional heating [3]. The equation for the change of the average plasma temperature in the high-frequency pulsation field has a form analogous to that used in [3] for heating by a monochromatic wave:

$$\frac{3}{2} \frac{\partial}{\partial t} T_{0e} = \frac{e^2}{m_e} \nu_e \int \frac{|E_\mathbf{k}|^2 \, d\mathbf{k}}{(\omega^+)^2} - 3 \frac{m_e}{m_i} \nu_e (T_{0e} - T_{0i}) \tag{2.8}$$

$$\frac{3}{2} \frac{\partial}{\partial t} T_{0i} = 3 \frac{m_e}{m_i} \nu_e (T_{0e} - T_{0i}) \tag{2.9}$$

According to [3] (this follows also from (2.8), (2.9)) during the time $\tau \sim (1/\nu_e)$ the electron temperature increases only by

$$\Delta T_{0e} \approx T_{0e} \left(\frac{\omega_{0e}}{\omega^+}\right)^2 \frac{W}{n_0 T_{0e}} \ll T_{0e}$$

since by assumption

$$(W / n_0 T_{0e}) \ll 1, \quad \omega^+ \gtrsim \omega_{0e}$$

The ion temperature increase is

$$\Delta T_{0i} \approx 3 \frac{m_e}{m_i} \Delta T_{0e}$$

After the time $\tau = \tau_1 = (m_i/m_e \nu_e)$ the difference of the average electron and ion temperatures approaches a plateau

$$\frac{T_{0e} - T_{0i}}{T_{0e}} \approx \frac{1}{3} \frac{m_i}{m_e} \left(\frac{\omega_{0e}}{\omega^+}\right)^2 \frac{W}{n_0 T_{0e}}$$

and the ion temperature increases by

$$\frac{\Delta T_{0i}}{T_{0e}} \approx \frac{m_i}{m_e} \left(\frac{\omega_{0e}}{\omega^+}\right)^2 \frac{W}{n_0 T_{0e}}$$

We denote

$$W_p = \frac{m_e}{m_i} n_0 T_{0e} \left(\frac{\omega^+}{\omega_{0e}}\right)^2 = \frac{E_p^2}{4\pi}$$

Here E_p is the so-called plasma field, and W_p is the plasma field energy density. Thus, if $W \ll W_p$, the electron and ion heating effect can be ignored. If the plasma is weakly ionized, the collisions of the ions with the neutrals may not permit them to raise the temperature sufficiently high, and then the ion temperature can be considered constant and equal to T, and the electron temperature ($\tau \gg \tau_1$) can be considered equal to $T(1 + W/W_p)$. For $W \gg W_p$ we have $T_{0e} \gg T_{0i}$. In such a system turbulent heating as a result of nonlinear excitation of ion sound is possible.

Let us return to (2.1)-(2.5). We find the solution of this set of equations for $V^{(2)}$, $n^{(2)}$, $T^{(2)}$, and $V^{(1)}$, $n^{(1)}$, $T^{(1)}$, with the aid of which we then find the nonlinear second-order polarizability with respect to the field $S_1(k, k_1, k_2)$, defined by the equality

$$j_k^* = \int S_1(k, k_1, k_2) E_{k_1}^+ E_{k_2}^+ \delta(k_1 + k_2 - k)\, dk_1 dk_2 \tag{2.10}$$

Let us evaluate the terms appearing in the momentum transport equation (2.2). To find S_1 we must know $V^{(1)}$, $n^{(1)}$, $T^{(1)}$, corresponding to the high frequency ω^+; by discarding the dissipative terms for the high frequency, we can obtain

$$n_{k_1}^{(1)} = n_0 \frac{k_1 V_{k_1}^{(1)}}{\omega_1}, \quad V_{k_1}^{(1)} = -i \frac{eE_{k_1}}{m_e \omega_1}, \quad T_{k_1}^{(1)} = 1.14 \frac{T_{0e} k_1 V_{k_1}^{(1)}}{\omega_1} \tag{2.11}$$

Here we consider the field E_k to be longitudinal for simplicity. Using (2.11), it is not difficult to show that the sum

$$m_e n_e \frac{\partial}{\partial t} \mathbf{V}_{e,\alpha}^{(1)} + e n_e^{(1)} E_\alpha = 0$$

The term $\partial n_e^{(1)} T_e^{(1)} / \partial x_\alpha$ in (2.2) can be shown with the aid of (2.11) to be $\sim E^2/(\omega^+)^4$, and therefore it can be neglected. Similarly, considering the expansion (2.7) and formula (2.11), it is easy to show that with the required accuracy

$$\pi_{e,\alpha\beta} = -0.73 \frac{n_0 T_{0e}}{\nu_e (T_{0e})} W_{\alpha\beta}^{(e)(2)}, \quad W_{\alpha\beta}^{(e)(2)} \sim V_e^{(2)}$$

and the magnitude of the friction force must be considered equal to

$$\mathbf{R} = -0.51\, m_e n_0 \nu_e \mathbf{U}^{(2)} - 0.71\, n_0 \frac{\partial}{\partial \mathbf{r}} T_e^{(2)}$$

The terms in the energy transport equation (2.4) can be evaluated similarly

$$n_e^{(1)} T_{0e} \frac{\partial}{\partial \mathbf{r}} V_e^{(1)} \sim \frac{E^2}{(\omega^+)^3}, \quad \mathbf{V}_e^{(1)} \frac{\partial}{\partial \mathbf{r}} T_e^{(1)} \sim \frac{E^2}{(\omega^+)^3}, \quad n_0 T_e^{(1)} \frac{\partial}{\partial \mathbf{r}} \mathbf{V}_e^{(1)} \sim \frac{E^3}{(\omega^+)^3}$$

$$\mathbf{q}_e = 0.71\, n_0 T_{0e} \mathbf{U}^{(2)} - 3.16 \frac{n_0 T_{0e}}{m_e \nu_e} \frac{\partial}{\partial \mathbf{r}} T_e^{(2)}, \qquad \frac{V_e^{(1)}}{V_i^{(1)}} = \frac{m_i}{m_e}$$

$$Q_e = -(\mathbf{R}^{(1)} \mathbf{U}^{(1)}) - 3 \frac{m_e}{m_i} n_0 \nu_e (T_e^{(2)} - T_i^{(2)}) = m_e n_0 \nu_e \mathbf{V}_e^{(1)} \mathbf{V}_e^{(1)} - 3 \frac{m_e}{m_i} n_0 \nu_e (T_e^{(2)} - T_i^{(2)})$$

Finally, the quantity

$$\pi_{e,\alpha\beta}^{(1)} \frac{\partial V_{e,\alpha}^{(1)}}{\partial x_\beta} \sim \frac{k_1^2 v_{Te}^2}{\nu_e^2} Q_e \quad \text{for} \quad k_1 v_{Te} \ll \nu_e$$

will be small.

Evaluating similarly the terms appearing in the ion transport equation, we finally obtain the equations for the corrections of second order in E:

$$\left(-i\omega + i \frac{k^2 v_{Te}^2}{\omega}\right) V_{ke}^{(2)} = -0.51\, \nu_e U_k^{(2)} - 1.71 ik v_{Te}^2 \frac{T_{ke}^{(2)}}{T_{0e}} - \frac{eE_k}{m_e} \tag{2.12}$$

$$\left(-\frac{3}{2} i\omega + 3.16 \frac{k^2 v_{Te}^2}{\nu_e} + 3 \frac{m_e}{m_i} \nu_e\right) \frac{T_{ke}^{(2)}}{T_{0e}} = -ikV_{ke}^{(2)} - 0.71 ikU_k^{(2)} +$$

$$+ 3 \frac{m_e}{m_i} \nu_e \frac{T_{ki}^{(2)}}{T_{0e}} + \frac{m_e \nu_e}{T_{0e}} \int \mathbf{V}_{k_1}^{(1)} \mathbf{V}_{k_2}^{(1)} d\lambda, \left(-i\omega + 1.28 \frac{k^2 v_{Ti}^2}{\nu_i} + i \frac{k^2 v_{Ti}^2}{\omega}\right) V_{ki}^{(2)}$$

$$= -ik v_{Ti}^2 \frac{T_{ki}^{(2)}}{T_{0i}} + 0.71 ik v_{Ti}^2 \frac{T_{ke}^{(2)}}{T_{0i}} + 0.51 \frac{m_e}{m_i} \nu_e U_k^{(2)} + \frac{eE_k}{m_i}, \left(-\frac{3}{2} i\omega + 3.9 \frac{k^2 v_{Ti}^2}{\nu_i} + 3 \frac{m_e}{m_i} \nu_e\right) \frac{T_{ki}^{(2)}}{T_{0i}}$$

$$= -ikV_{ki}^{(2)} + 3 \frac{m_e}{m_i} \nu_e \frac{T_{ke}^{(2)}}{T_{0i}}, \; d\lambda = \delta(k - k_1 - k_2)\, dk_1 dk_2, \qquad k = \{\mathbf{k}, \omega\}$$

Solving this system, we obtain

$$V_{ke}^{(2)} = -\frac{ik\nu_e A_e}{\varkappa \Omega_e \omega_e} \int V_{k_1}^{(1)} V_{k_2}^{(1)} d\lambda, \qquad V_{ki}^{(2)} = \frac{ik\nu_e A_i m_e}{\varkappa \Omega_e \omega_i m_i} \int V_{k_1}^{(1)} V_{k_2}^{(1)} d\lambda \tag{2.13}$$

Here

$$\omega_i = -i\omega + i \frac{k^2 v_{Ti}^2}{\omega} + 1.28 \frac{k^2 v_{Ti}^2}{\nu_i} + \frac{k^2 v_{Ti}^2}{\Omega_i} - \left(0.71 - \frac{\delta\nu}{\Omega_i}\right)\left(1 + \frac{\delta\nu}{\Omega_i}\right) \frac{T_{0e}}{T_{0i}} \frac{k^2 v_{Ti}^2}{\Omega_e} \tag{2.14}$$

$$\omega_e = -i\omega + i \frac{k^2 v_{Te}^2}{\omega} + 1.71 \frac{k^2 v_{Te}^2}{\Omega_e}\left(1 + \frac{\delta\nu}{\Omega_i}\right), \qquad \delta\nu = 3 \frac{m_e}{m_i} \nu_e$$

$$\Omega_i = -\frac{3}{2} i\omega + \delta\nu + 3.9 \frac{k^2 v_{Ti}^2}{\nu_i}, \quad \Omega_e = -\frac{3}{2} i\omega + \delta\nu + 3.16 \frac{k^2 v_{Te}^2}{\nu_e} - \frac{(\delta\nu)^2}{\Omega_i}$$

$$\varkappa = 1 + \varkappa^\circ \left(\frac{1}{\omega_e} + \frac{m_e}{m_i} \frac{1}{\omega_i}\right), \quad \varkappa^\circ = 0.51 \nu_e + 1.71 \left(0.71 - \frac{\delta\nu}{\Omega_i}\right) \frac{k^2 v_{Te}^2}{\Omega_e}$$

$$A_e = 1.71 + \frac{m_e}{m_i} \frac{\varkappa^\circ}{\omega_i}\left(1 + \frac{\delta\nu}{\Omega_i}\right), \qquad A_i = 0.71 - \frac{\delta\nu}{\Omega_i} - \frac{\varkappa^\circ}{\omega_e}\left(1 + \frac{\delta\nu}{\Omega_i}\right)$$

The sought expression for S_1 has the form

$$S_1(k, k_1, k_2) = -i \frac{en_0 k \nu_e}{\varkappa \Omega_e}\left(\frac{A_e}{\omega_e} + \frac{m_e}{m_i} \frac{A_i}{\omega_i}\right) \frac{V_{k_1}^{(1)} V_{k_2}^{(1)}}{E_{k_1} E_{k_2}} \sim V_{ke}^{(2)} - V_{ki}^{(2)} \tag{2.15}$$

For $\omega \gg \delta\nu$, kv_{Ti} the result (2.15) coincides with that obtained in [1]. Thus, (2.13) generalizes the results of [1] to the case $\omega \lessdot \delta\nu$ and $\omega \lessdot kv_{Ti}$.

The nonlinear current S_2, defined by the relation

$$j_k^+ = \int S_2(k, k_1, k_2) E_{k_1}^+ E_{k_2}^* d\lambda \tag{2.16}$$

has formally the same forms as in [1]

$$S_2(k, k_1, k_2) = \frac{ie^3 n_0 k_2 (\mathbf{k}\mathbf{k}_1)}{m_e^2 \omega \varkappa \omega_e \omega_2 k k_1} \tag{2.17}$$

where, however, $\varkappa$, ω_e are defined by (2.14). Finally, the nonlinear third-order current is connected with the second-order current by the relation found in [1].

3. We shall present an example of the calculation of the spectral transfers of Langmuir waves if the difference of their frequencies is less than $\delta\nu$. A simple calculation yields

$$\gamma_{\mathbf{k}} = \frac{1}{|E_{\mathbf{k}}|^2}\frac{\partial}{\partial t}|E_{\mathbf{k}}|^2 = -\frac{1.71\nu_e n_0 e^4}{m_i m_e^2 \omega_{0e}^3}\,\mathrm{Re}\int \frac{|\mathbf{k}-\mathbf{k}_1|^2(\mathbf{k}\mathbf{k}_1)^2}{\omega_e \varkappa \Omega_e k^2 k_1^2}\,\frac{\omega_e |E_{\mathbf{k}_1}|^2 d\mathbf{k}_1}{\omega_i(\omega-\omega_1)[1+m_e\omega_e/m_i\omega_i]} \tag{3.1}$$

Most effective is the interaction of waves whose frequency difference is close to the speed $\omega_s = \sqrt{{}^{10}\!/_{3}}\,kv_{Ti}$ of sound vibrations or, more precisely, if

$$\frac{\Delta\omega-\omega_s}{\Delta\omega} < \frac{|\mathbf{k}_1-\mathbf{k}_2|^2 v_{Te}^2}{\Delta\omega\nu_e}, \qquad |\Delta\omega-\omega_s| \gg \gamma_s \tag{3.2}$$

Here $\Delta\omega=\omega_1-\omega_2$ is the difference of the interacting wave frequencies. We obtain the estimate

$$\gamma_{\mathbf{k}} \approx -\frac{(\Delta\omega-\omega_s)\,\omega_{0e}}{(kv_{Te}/\nu_e)^4\nu_e}\,\frac{W}{n_0 T_{0e}} \tag{3.3}$$

If $\Delta\omega-\omega_s$ is of order $(k^2 v_{Te}^2/\nu_e)$, then γ_k is of order

$$\gamma_{\mathbf{k}} \approx \frac{\omega_{0e}\nu_e^2}{k^2 v_{Te}^2}\,\frac{W}{n_0 T_{0e}} \tag{3.4}$$

Formula (3.4) shows that such interactions are very effective. However, in using this relation we must bear in mind that $W/n_0T_{0e} \ll (m_e/m_i)$ and, moreover, only those waves having very close frequencies $\Delta\omega \sim \omega_s \ll (\nu_e m_e/m_i)$ interact intensely. The actual spectra of the Langmuir vibrations in a turbulent plasma may be considerably broader. Therefore, we are discussing here transfer of the "cascade" type (see [5]).

4. Let us examine as another example the possibility of the decay of high-frequency waves (for example, transverse laser waves under conditions of laser sparking) into low-frequency sound $\omega_s \ll (\nu_e m_e/m_i)$. It is not difficult to see that the dispersion equation describing this process has the standard from [4, 5]

$$\omega' + i\gamma_s = -\frac{1}{(2\pi)^4}\int \frac{u(\mathbf{k},\mathbf{k}_1)(N^t_{\mathbf{k}_1} - N^t_{\mathbf{k}_1-\mathbf{k}})}{\omega' + \Delta\omega_{\mathbf{k}\mathbf{k}_1} + i\delta}\,d\mathbf{k}_1 \tag{4.1}$$

$$\Delta\omega_{\mathbf{k}\mathbf{k}_1} = -\mathrm{Re}\,(\omega^t_{\mathbf{k}_1} - \omega^t_{\mathbf{k}_1-\mathbf{k}} - \omega^s_{\mathbf{k}})$$

Here ω' is the nonlinear correction to the sound vibration frequency $u(\mathbf{k}, \mathbf{k}_1)$, which is usually the probability of the decay process in question (in the collisionless case), and in the present case is proportional to the product of the nonlinear polarizabilities S_1 and S_2

$$u(\mathbf{k},\mathbf{k}_1) = (2\pi)^4 \frac{S_1(\mathbf{k},\,\omega^t_{\mathbf{k}_1}-\omega^t_{\mathbf{k}_1-\mathbf{k}};\ \mathbf{k}_1,\,\omega^t_{\mathbf{k}_1};\ \mathbf{k}-\mathbf{k}_1,\,-\omega^t_{\mathbf{k}_1-\mathbf{k}})}{(\partial\omega\varepsilon^s(\omega,\mathbf{k})/\partial\omega)_1(\partial\omega^2\varepsilon^t(\omega,\mathbf{k}_1)/\partial\omega)_2} \tag{4.2}$$

$$\times\, 16\,(\omega^t_{\mathbf{k}_1-\mathbf{k}} + \mathrm{Re}\,\omega_{\mathbf{k}}^s)\,\frac{S_2(\mathbf{k}_1,\,\omega^t_{\mathbf{k}_1-\mathbf{k}}+\mathrm{Re}\,\omega^s_{\mathbf{k}};\ \mathbf{k}_1-\mathbf{k},\,\omega^t_{\mathbf{k}_1-\mathbf{k}};\ \mathbf{k},\,\mathrm{Re}\,\omega^s_{\mathbf{k}})}{(\partial\omega^2\varepsilon^t(\omega,\mathbf{k}_1-\mathbf{k})/\partial\omega)_3}$$

Here the subscripts 1, 2, 3 on the parentheses denote the corresponding substitutions of the values of ω

$$\omega = \mathrm{Re}\,\omega_{\mathbf{k}}^s, \quad \omega = \omega_{\mathbf{k}_1}{}^t, \quad \omega = \omega^t_{\mathbf{k}_1-\mathbf{k}}$$

Usually kinetic nonlinear instability occurs when $\Delta\omega_{\mathbf{k}\mathbf{k}_1} > \omega'$ (see [4, 5]) and the integrand is proportional to $(-i\pi\delta(\Delta\omega_{\mathbf{k}\mathbf{k}_1}))$. In the case $\omega_{\mathbf{k}}^s < (\nu_e m_e/m_i)$ it follows from (2.15) and (2.17) that

$$u(\mathbf{k},\mathbf{k}_1) = \frac{1.71\,(2\pi)^2\nu_e\omega_{0e}^4}{8\,\omega^t_{\mathbf{k}_1}\omega^t_{\mathbf{k}_1-\mathbf{k}}4\pi n_0 T_{0e}}\left(1+\frac{(\mathbf{k}_1,\,\mathbf{k}_1-\mathbf{k})^2}{k_1^2|\mathbf{k}_1-\mathbf{k}|^2}\right) \tag{4.3}$$

In obtaining (4.3) we dropped small terms of order $(k^2 v_{Te}^2/\nu_e\omega_{\mathbf{k}}^s)$. Consequently $(\mathrm{Re}\,u/\mathrm{Im}\,u) \sim (\gamma_s/\omega_{\mathbf{k}}^s)$, and therefore excitation of low-frequency acoustic vibrations of the decay type is not possible (see [4]).

If the transverse wave beam has some frequency scatter $\Delta\omega_1$ and angle scatter $\Delta\mathbf{k}_1$, we can assume that for change of ω_1 and $\mathbf{k}_1$ in the indicated intervals the maximal value of $\Delta\omega_{\mathbf{k}\mathbf{k}_1}$ is Δ_1 and $\max(\Delta\omega_{\mathbf{k}\mathbf{k}_1+k}) = \Delta_2$. We denote the corresponding minimal values by δ_1 and δ_2. If δ_1, Δ_1 and δ_2, Δ_2 are larger than Re $\omega_{\mathbf{k}}^s\lambda$, the new form of nonlinear dissipative instability being considered appears.

Let δ_2, $\Delta_2 > \delta_1, \Delta_1$, then

$$\omega' + i\gamma_s = i\frac{1.71\nu_e\omega_{0e}^4}{(2\pi)^3 32\pi n_0 T_{0e}}\int a(\mathbf{k}, \mathbf{k}_1)\frac{N^t_{\mathbf{k}_1}}{\Delta\omega_{d\mathbf{k}_1}}d\mathbf{k}_1 \tag{4.4}$$

$$a(\mathbf{k}, \mathbf{k}_1) = \frac{1}{\omega^t_{\mathbf{k}_1}\omega^t_{\mathbf{k}_1-\mathbf{k}}}\left(1 + \frac{(\mathbf{k}_1, \mathbf{k}_1 - \mathbf{k})^2}{\mathbf{k}_1^2|\mathbf{k}_1 - \mathbf{k}|^2}\right), \quad \Delta\omega_{\mathbf{k}_1} = \omega^t_{\mathbf{k}_1} - \omega^t_{\mathbf{k}_1-\mathbf{k}}$$

or

$$\omega' + i\gamma_s = -i\frac{1.71\nu_e\omega_{0e}^4}{(2\pi)^3 32\pi n_0 T_{0e}}\int\left\{\left(\mathbf{k}\frac{\partial a(\mathbf{k}, \mathbf{k}_1)}{\partial \mathbf{k}_1}\right) - \frac{a(\mathbf{k}, \mathbf{k}_1)}{\Delta\omega_{\mathbf{k}_1}}\left(\mathbf{k}\frac{\partial\Delta\omega_{\mathbf{k}_1}}{\partial\mathbf{k}_1}\right)\right\}N^t_{\mathbf{k}_1}\frac{d\mathbf{k}_1}{\Delta\omega_{\mathbf{k}_1}} \tag{4.5}$$

for δ_2, $\Delta_2 \sim \delta_1$, Δ_1, $\mathbf{k}_1 \gg \mathbf{k}$.

Formulas (4.4) and (4.5) make it possible to evaluate the nonlinear excitation increments of the low-frequency acoustic waves propagating at an acute angle to the transverse wave beam. The estimate using (4.4) yields

$$\gamma \sim \frac{\omega_{0e}}{16}\left(\frac{\omega_{0e}}{\omega^t}\right)^3\frac{\nu_e}{kc}\frac{W^t}{n_0 T_{0e}} \qquad \text{for} \quad \frac{k}{k_1} \gg \frac{v_{Ti}}{c} \tag{4.6}$$

Assuming that $\nu_e m_e / m_i$, $\omega_k^s > \gamma > \gamma_s$, we obtain the conditions under which the excitation (4.6) is possible

$$\left\{\frac{m_e}{m_i}, \frac{kv_{Ti}}{\nu_e}\right\} > \frac{\omega_{0e}}{kc}\left(\frac{\omega_{0e}}{\omega^t}\right)^3\frac{W^t}{n_0 T_{0e}} > \frac{k^2 v_{Te}^2}{\nu_e^2}$$

The estimates from (4.5) can be obtained similarly.

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